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4-REGULAR INTEGRAL GRAPHS AVOIDING ±3 IN THE SPECTRUM

Dragan Stevanović

We determine all connected 4-regular integral graphs avoiding ±3 in the spectrum. There are exactly 16 bipartite and 8 nonbipartite such graphs. The smallest bipartite one is $K_{4,4}$, while the largest has 32 vertices. Among these graphs there are two triplets of cospectral nonisomorphic graphs and two pairs of cospectral nonisomorphic graphs. The smallest nonbipartite one is K_5, and the largest has 15 vertices. Among these graphs there is a pair of cospectral nonisomorphic graphs.

1. INTRODUCTION

A simple graph is called *integral* if all eigenvalues of its adjacency matrix are integers. The quest for integral graphs was initiated by Harary and Schwenk in [**10**]. All thirteen connected cubic integral graphs were obtained by Cvetković and Bussemaker in [**4**] and [**2**], and independently by Schwenk in [**12**]. In fact, Cvetković [**4**] proved that the set of connected regular integral graphs of a fixed degree is finite. Similarly, the set of connected integral graphs with bounded vertex degrees is finite. Radosavljević and Simić in [**13**] determined all thirteen nonregular nonbipartite connected integral graphs whose maximum degree equals four. A survey of results on integral graphs may be found in [**1**].

Here we are interested in connected 4-regular integral graphs. This paper is based on author's Ph.D. thesis [**14**].

The search for integral graphs becomes easier if we use product of graphs. Given two graphs G and H, with vertex sets $V(G)$ and $V(H)$, their product $G \times H$ is the graph with vertex set $V(G) \times V(H)$ in which two vertices (x, a) and (y, b) are adjacent if and only if x is adjacent to y in G and a is adjacent to b in H. If $\lambda_1, \dots, \lambda_n$ are the eigenvalues of G, and $\mu_1, \dots, \mu_m$ are the eigenvalues of H, then the eigenvalues of $G \times H$ are $\lambda_i \mu_j$ for $i = 1, \dots, n,\ j = 1, \dots, m$ (see [**6**]). If graph

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G is nonbipartite, 4-regular and integral, then the product $G \times K_2$ is connected, bipartite, 4-regular and integral, since the eigenvalues of K_2 are 1 and -1. Therefore, in determining 4-regular integral graphs we can consider bipartite graphs only, and later extract such nonbipartite graphs from the decompositions of bipartite ones in the form $G \times K_2$.

Suppose that G is a 4-regular bipartite integral graph with $p = 2n$ vertices. Using superscripts to represent multiplicities, we may write its spectrum in the form

$$4, 3^x, 2^y, 1^z, 0^{2w}, -1^z, -2^y, -3^x, -4.$$

Let q and h denote the numbers of quadrilaterals and hexagons in G. Possible spectra for such graphs are determined in [**9**]. Due to the huge number of possible spectra, it is unlikely that all integral 4-regular graphs will be soon determined. For example, there are 1259 possible spectra for a 4-regular integral graph in which each of the integers $-4, \dots, 4$ occurs as an eigenvalue. In one of the cases the potential graph has 5040 vertices. Therefore, in this paper we shall consider a more modest task, i.e. to determine graphs without 3 and -3 in the spectrum, i.e. with $x = 0$.

It is well known [**6**] that the sum of k^{th} powers of the eigenvalues is just the number of closed walks of length k. On the other hand, for $k = 0, 2, 4$ and 6 the number of closed walks of length k in a 4-regular graph is expressible in terms of n, q and h. Using these facts, we form the Diophantine equations

$$\begin{aligned}
\frac{1}{2}\sum \lambda_i^0 &= 1 + x + y + z + w = n,\\
\frac{1}{2}\sum \lambda_i^2 &= 16 + 9x + 4y + z = 4n,\\
\frac{1}{2}\sum \lambda_i^4 &= 256 + 81x + 16y + z = 28n + 4q,\\
\frac{1}{2}\sum \lambda_i^6 &= 4096 + 729x + 64y + z = 232n + 72q + 6h,
\end{aligned}$$

from which in the case $x = 0$ easily follows that either $(z, w) = (0, 3)$, $n \leq 16$ or $(z, w) = (4, 0)$, $n \leq 15$. From the HOFFMAN's identity (Lemma 1) we get further conditions on n. The spectra with $x = 0$, obtained in this way in [**9**], are shown in Table **??**.

2. TRIVIAL CASES

A 4-regular bipartite integral graph with at most 12 vertices (at most 6 in each bipartite class) may be considered trivial, since its bipartite complement is regular with vertex degree at most 2.

For $n = 4$ we get the complete bipartite graph $D_1 = K_{4,4}$ (Fig. 1) which is integral with the spectrum is $4, 0^6, -4$.

For $n = 5$ the bipartite complement of G is 1-regular, i.e. it is $\overline{5K_2}$. Then G is integral with the spectrum $4, 1^4, -1^4, -4$ and it is D_2 from Fig. 1.

n	x	y	z	w	q	h
4	0	0	0	3	36	96
6	0	2	0	3	30	112
8	0	4	0	3	24	128
12	0	8	0	3	12	160
16	0	12	0	3	0	192

n	x	y	z	w	q	h
5	0	0	4	0	30	130
6	0	1	4	0	27	138
9	0	4	4	0	18	162
10	0	5	4	0	15	170
12	0	7	4	0	9	186
15	0	10	4	0	0	210

Table 1: Possible integral graph spectra with $(x, z, w) = (0, 0, 3)$ and $(x, z, w) = (0, 4, 0)$.

For $n = 6$ the bipartite complement $\overline{G}^b$ of G is 2-regular, so it is union of cycles. There are several possibilities:

(i) $\overline{G}^b \cong 3C_4$. The spectrum of G is $4, 2^2, 0^6, -2^2, -4$ and it is graph D_3 from Fig. 1;

(ii) $\overline{G}^b \cong C_4 \cup C_8$. The spectrum of G is $4, 2, \sqrt{2}^2, 0^4, -\sqrt{2}^2, -2, -4$, thus it is not integral;

(iii) $\overline{G}^b \cong 2C_6$. The spectrum of G is $4, 2, 1^4, -1^4, -2, -4$ and it is shown as D_4 in Fig. 1;

(iv) $\overline{G}^b \cong C_{12}$. The spectrum of G is $4, \sqrt{3}^2, 1^2, 0^2, -1^2, -\sqrt{3}^2, -4$, thus it is not integral.

3. HOW TO ATTACK NONTRIVIAL CASES?

Here we present results used to determine nontrivial integral graphs. Consider the general case of an r-regular bipartite graph $G = (U, V, E)$. Let G have distinct eigenvalues $r = \lambda_1, \lambda_2, \ldots, \lambda_k$. Let A be the adjacency matrix of G, J is the matrix all of whose entries are 1, while I is the identity matrix.

Lemma 1. (Hoffman [**11**]) *The adjacency matrix A satisfies the equation*

$$\prod_{i=2}^{k}(r - \lambda_i)J = n\prod_{i=2}^{k}(A - \lambda_i I). \tag{1}$$

Let $n_{u,v}$ denote the number of common neighbors of vertices $u, v \in V$. The number of walks of length 2 starting from the vertex u of a regular graph of degree r is equal to r^2. On the other hand it is also equal to $\sum_{v \in V} n_{u,v}$. Since $n_{u,u} = r$

we get that

$$\sum_{v\in V-\{u\}} n_{u,v} = r(r-1). \tag{2}$$

For the regular graph with 5 (resp. 6) distinct eigenvalues Lemma 1 yields that the polynomial in A in the right-hand side of identity (1) is of degree 4 (resp. 5). Considering (u,u)-entries on both sides of the identity, taking into account that in the case of bipartite graphs $(A^{2k-1})_{u,u} = 0, k \in \mathbf{N}$, we see that for some a_0, a_1, a_2 (that depend only on $r = \lambda_1, \ldots, \lambda_k$) the following equality holds

$$a_0(A^4)_{u,u} + a_1(A^2)_{u,u} + a_2 = \prod_{i=2}^{k}(r-\lambda_i).$$

Now $(A^2)_{u,u} = r$ and therefore the number $(A^4)_{u,u}$ does not depend on u. Further, since $(A^4)_{u,u}$ denotes the number of closed walks of length 4 that start from u it follows that $(A^4)_{u,u} = r^2 + r(r-1) + 2q_u$, where q_u denotes the number of quadrilaterals to which the vertex u belongs. An important consequence we use, is that q_u does not depend on u, and therefore $q_u = \frac{2q}{n}$. On the other hand, from $(A^4)_{u,u} = \sum_{v\in V} n_{u,v}^2$ and $n_{u,u} = r$ follows

$$\sum_{v\in V-\{u\}} n_{u,v}^2 = r(r-1) + 2q_u. \tag{3}$$

Besides these results, we are also using the famous CAUCHY's interlacing theorem.

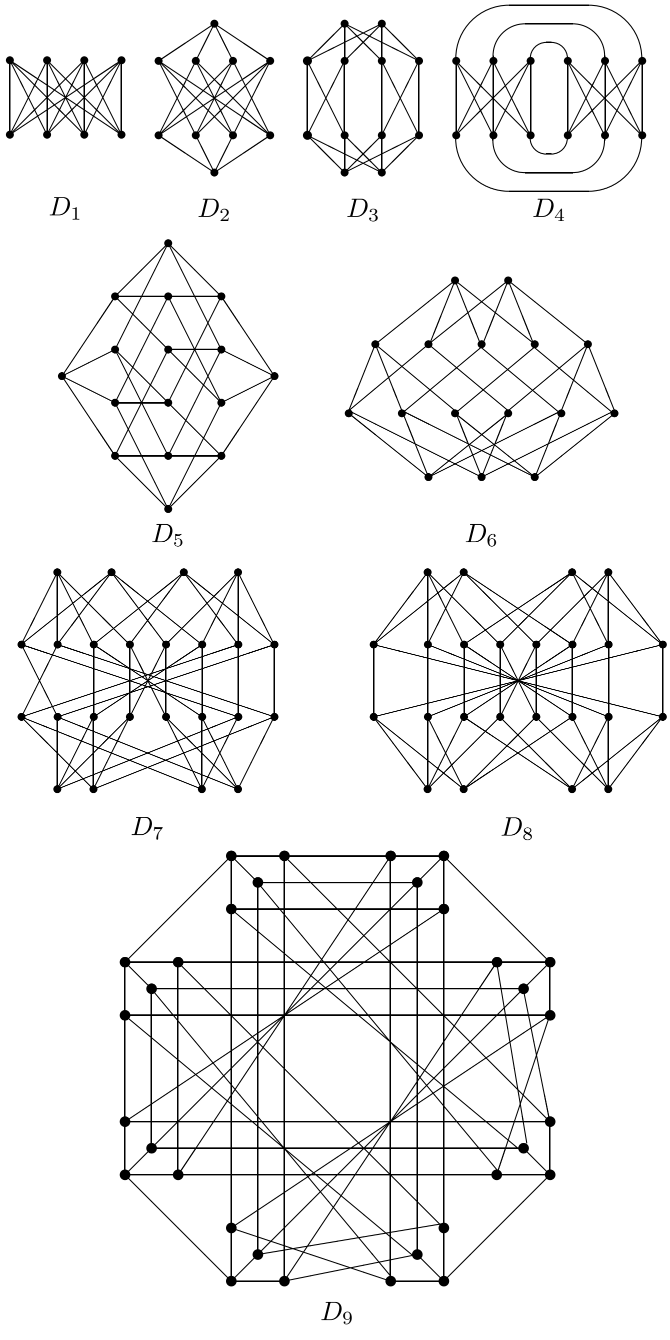
Theorem 2. (see, for example, [**6**], p. 19) *Let I be the induced subgraph of a graph H. Let the eigenvalues of H are $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n$, while the eigenvalues of I are $\mu_1 \geq \mu_2 \geq \ldots \geq \mu_m$. Then $\lambda_i \geq \mu_i \geq \lambda_{n-m+i}$ $(i = 1, \ldots, m)$.*

The method we use to determine nontrivial integral graphs is as follows. Using q_u and the equations (2) and (3) (with $r = 4$) we get a few possibilities (in most cases only one) for the family of numbers $\{n_{u,v} | v \in V - \{u\}\}$ for $u \in V$. This helps to construct a large part of a hypothetical integral graph. The number of further possibilities is reduced by showing that certain of these subgraphs are incoherent with the interlacing theorem, and the following theorem may be proved:

Theorem 3. [**14**] *There are* 16 *bipartite* 4*-regular integral graphs avoiding* ± 3 *in the spectrum, shown in Figs.* 1 *and* 2.

Complete proof of this theorem is given in author's Ph.D. thesis [**14**]. As an illustration, we give proofs for the cases $(x,y,z,w) = (0,4,0,3)$ and $(x,y,z,w) = (0,10,4,0)$ in the appendix.

Among D_1–D_{16} there are two triplets of cospectral graphs: (D_{10}, D_{11}, D_{12}) with 18 vertices and (D_{13}, D_{14}, D_{15}) with 20 vertices; and two pairs of cospectral graphs: (D_5, D_6) with 16 vertices and (D_7, D_8) with 24 vertices.



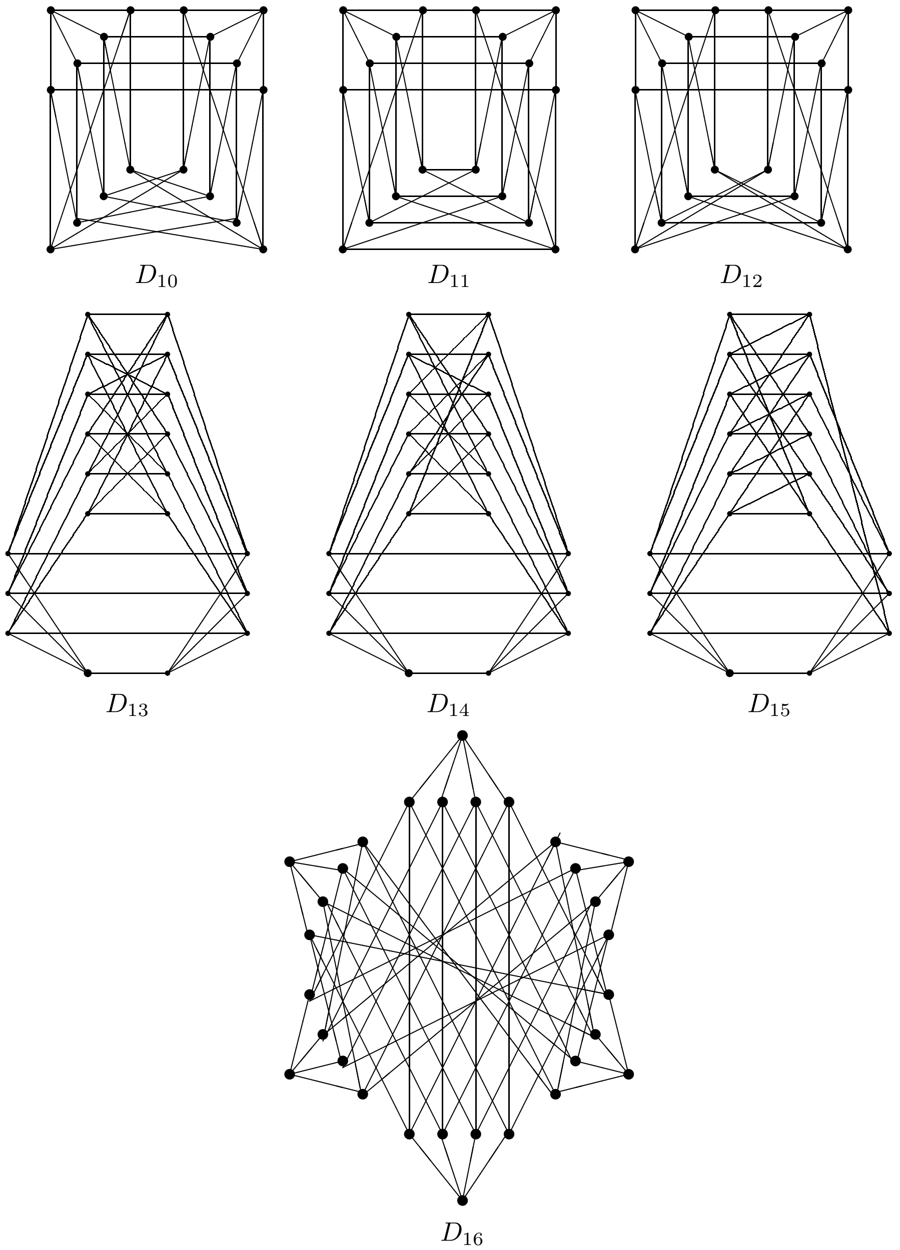
D_1 D_2 D_3 D_4

D_5 D_6

D_7 D_8

D_9

Figure 1: 4-Regular bipartite integral graphs.



D_{10} D_{11} D_{12}

D_{13} D_{14} D_{15}

D_{16}

Figure 2: 4-Regular bipartite integral graphs (continued).

4. NONBIPARTITE GRAPHS

If G is a nonbipartite 4-regular integral graph avoiding ± 3 in its spectrum, then $G \times K_2$ is still 4-regular, integral, avoids ± 3 in the spectrum, but it is bipartite and thus it is one of the graphs $D_1 - D_{16}$. It is therefore sufficient to find out which of these graphs can be decomposed in the form $G \times K_2$. The following lemma, whose proof is trivial, and an observation following it are taken from [**12**].

Lema 4. *A bipartite graph* $B = (U, V, E)$ *can be decomposed as a product* $G \times K_2$ *if and only if there is a bijection* $f\colon U \to V$ *such that* u *is not adjacent to* $f(u)$ *and if* u *is adjacent to* $f(v)$ *then* v *is adjacent to* $f(u)$*, for each* $u, v \in U$.

Furthermore, we observe that if

$$u = u_0, f(u_1), u_2, f(u_3), u_4, \ldots, u_{2n}, f(u_{2n+1}) = f(u)$$

is a path joining u to $f(u)$, then Lemma 4 requires that

$$f(u) = f(u_0), u_1, f(u_2), u_3, f(u_4), \ldots, f(u_{2n}), u_{2n+1} = u$$

is a second path from u to $f(u)$. This path is necessarily different, for if they were the same we would be forced to have u_n joined to $f(u_n)$ contrary to Lemma 4. This observation leads to the conclusion that for any u, an even number of shortest paths of odd length $2k+1$ must join u and $f(u)$.

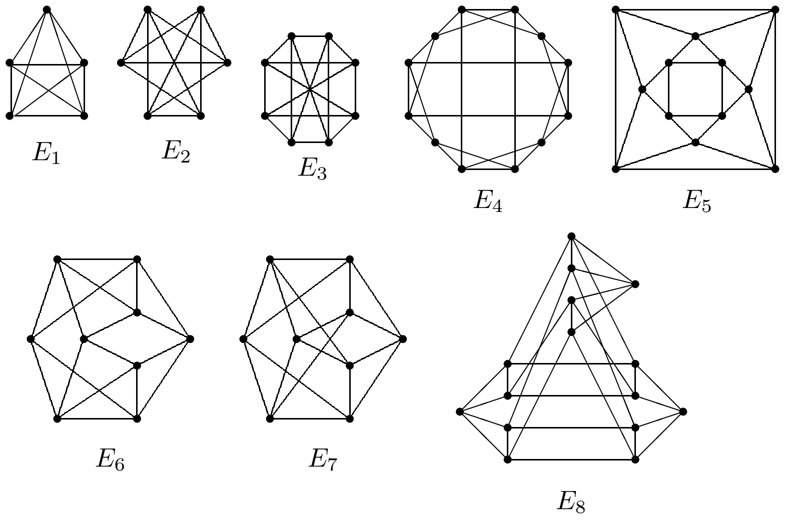


Figure 3: Nonbipartite 4-regular integral graphs

Using these two results, the following theorem may be proved:

Theorem 5. *There are* 8 *nonbipartite* 4*-regular integral graphs avoiding* ± 3 *in the spectrum, shown in Fig.* 3.

The complete proof of this theorem is given in author's Ph.D. thesis [**14**].

Among E_1–E_8 there is a pair (E_4, E_5) of cospectral graphs with 12 vertices, graphs E_2 and E_7 are strongly regular, while E_7 is also self-complementary. Graphs $E_1, \dots, E_8$ have the least eigenvalue equal to -2 and they are either line graphs or coctail-party graphs, except E_4 which is one of the graphs found in [**3**].

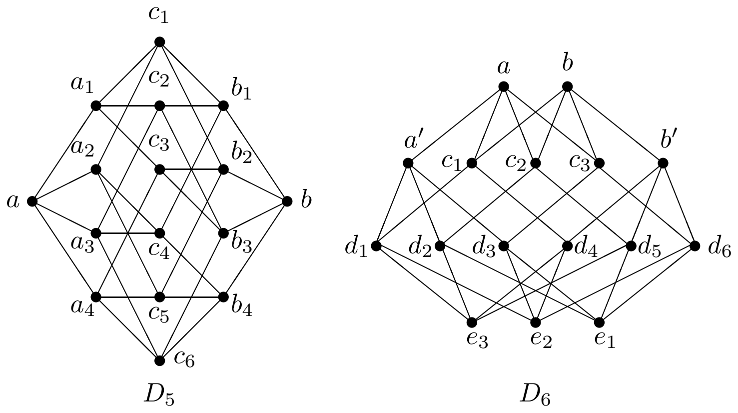
5. APPENDIX

5.1. Graphs with spectrum $4, 2^4, 0^6, -2^4, -4$. HOFFMAN's identity (1) reads now $24J = (A^4 - 4A^2) + 4(A^3 - 4A)$. From Table **??** we see that $q = 24$ and $q_u = 6, u \in V$. Therefore

$$\sum_{v \in V - \{u\}} n_{u,v}^2 = 24,$$

and the possibilities for the family $\{n_{u,v} | v \in V - \{u\}\}$ are (denoting the multiplicities with superscripts) $\{3, 2^3, 1^3\}$ and $\{2^6, 0\}$.

There are two cases to consider. Suppose that for every vertex u of G holds that $\{n_{u,v} | v \in V - \{u\}\} = \{2^6, 0\}$. Let a be an arbitrary vertex of G, and let b be the vertex for which $n_{a,b} = 0$. Denote the neighbors of a with $a_1, \dots, a_4$, and the neighbors of b with $b_1, \dots, b_4$. Denote with $c_1, \dots, c_6$ the remaining vertices of G. For any $k \neq l$ vertices a_k and a_l have a as the common neighbor, so they must have exactly one more common neighbor in the set $\{c_1, \dots, c_6\}$. Since $n_{a,c_i} = 2$ $(i = 1, \dots, 6)$, the vertex c_i $(i = 1, \dots, 6)$ is the common neighbor for exactly one pair of vertices from $\{a_1, \dots, a_4\}$. Further, for every a_i $(i = 1, \dots, 4)$ there is a vertex from $\{b_1, \dots, b_4\}$ that has no common neighbors with a_i, and we may suppose this vertex is b_{5-i}. We constructed the graph D_5 (Fig. 4), which is integral.



D_5 D_6

Figure 4: Integral graphs with spectrum $4, 2^4, 0^6, -2^4, -4$

Suppose now that there is a vertex a of G such that $\{n_{a,v} | v \in V - \{a\}\} = \{3, 2^3, 1^3\}$. Let $b \in V$ be such that $n_{a,b} = 3$, let $c_1, c_2, c_3 \in U$ be the common neighbors of a and b, and let $a' \in U$ be the remaining neighbor of a, while $b' \in U$ is the remaining neighbor of b. Let $d_1, \dots, d_6$ be the remaining vertices of V, while e_1, e_2, e_3 are the remaining vertices of U. Further, let $n_{a,d_1} = n_{a,d_2} = n_{a,d_3} = 2$ and $n_{a,d_4} = n_{a,d_5} = n_{a,d_6} = 1$.

From HOFFMAN's identity we get that

$$36 = A^4_{a,b} = 24 + \sum_{j=1}^{6} n_{a,d_j} \cdot n_{b,d_j},$$

where 24 holds for the number of walks of length 4 from a to b whose middle vertex is either a or b. Since further

$$\prod_{j=1}^{6} n_{a,d_j} \cdot n_{b,d_j} = 2^6,$$

the only possibility is that for $j = 1, \dots, 6$ it holds $n_{a,d_j} \cdot n_{b,d_j} = 2$, i.e. it must be $n_{b,d_1} = n_{b,d_2} = n_{b,d_3} = 1$, while $n_{b,d_4} = n_{b,d_5} = n_{b,d_6} = 2$.

Then it is obvious that each of d_1, d_2, d_3 is adjacent to a' and to a vertex from $\{c_1, c_2, c_3\}$, while each of d_4, d_5, d_6 is adjacent to b' and to a vertex from $\{c_1, c_2, c_3\}$. We may suppose that c_i is adjacent to d_i and d_{i+3}, $i = 1, 2, 3$. Vertex c_i $(i = 1, 2, 3)$ cannot have more than 2 common neighbors with any of the remaining vertices and therefore it must be $\{n_{c_i,u} | u \in U - \{c_i\}\} = \{2^6, 0\}$. We may further suppose that $n_{c_i,e_i} = 0$ $(i = 1, 2, 3)$ and the graph D_6 (Fig. 4) is then completely described.

So, we have proved

Proposition 1. *The only graphs with spectrum* $4, 2^4, 0^6, -2^4, -4$ *are* D_5 *and* D_6.

5.2. Graphs with spectrum $\mathbf{4, 2^{10}, 1^4, -1^4, -2^{10}, -4}$.

From the Table 1 in this case $q = 0$ and therefore $q_u = 0, u \in V$. It follows that

$$\sum_{v \in V - \{u\}} n^2_{u,v} = 12,$$

and the only possibility for the family $\{n_{u,v} | v \in V - \{u\}\}$ is $\{1^{12}, 0^2\}$.

Take an arbitrary vertex $a_1 \in V$. Then let $a_2, a_3 \in V$ be the vertices having no common neighbor with a_1. Denote the remaining vertices in the chromatic class of a_1 with $v_1, \dots, v_{12}$. Denote the neighbors of a_1 by u_i $(i = 1, 2, 3, 4)$. We may suppose that the vertex u_i $(i = 1, \dots, 4)$ is adjacent to v_i, v_{i+4}, v_{i+8}, since otherwise there would be two vertices u_k and u_l $(1 \le k < l \le 4)$ having a common neighbor v_j, which, together with a_1, form a quadrilateral.

We first show that $n_{a_2,a_3} = 1$ is impossible. Suppose so and denote the common neighbor of a_2 and a_3 by d. Then d must be adjacent to two of vertices $v_1, \dots, v_{12}$, that are not adjacent to the same vertex from $u_1, \dots, u_4$. But then, in a subgraph induced by $a_1, a_2, a_3, d, u_1, \dots, u_4, v_1, \dots, v_{12}$ the second largest eigenvalue is $\mu_2 = 2.0876$, which is contradictory to the interlacing theorem.

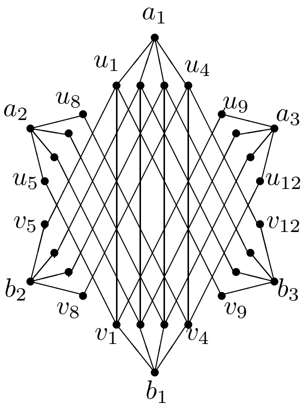


Figure 5: Partial construction of G

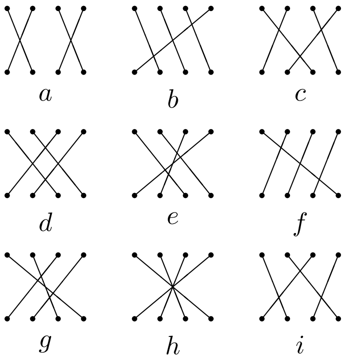


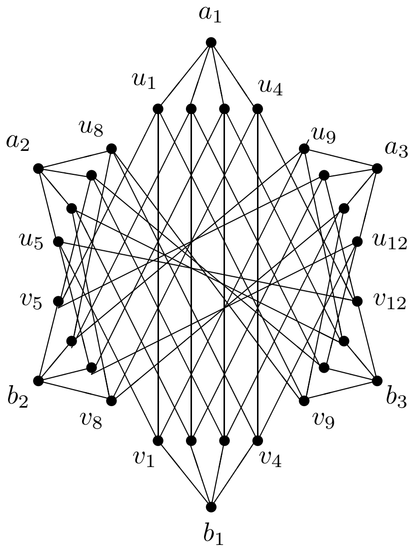
Figure 6: Possibilities for G_{kl}

Therefore $n_{a_2,a_3} = 0$. Denote the neighbors of a_2 by $u_5, \dots, u_8$, and the neighbors of a_3 by $u_9, \dots, u_{12}$. Denote by b_1, b_2 and b_3 the remaining three vertices of a graph. Since $n_{a_i,v_j} = 1$ $(i = 1, 2, 3; j = 1, \dots, 12)$, any of the vertices v_i $(i = 1, \dots, 12)$ is adjacent to exactly one of the vertices b_1, b_2, b_3. We may suppose that b_i $(i = 1, 2, 3)$ is adjacent to $v_{4i-3}, v_{4i-2}, v_{4i-1}, v_{4i}$. Also, we may suppose that v_j $(j = 1, \dots, 4)$ is adjacent to u_j, u_{j+4} and u_{j+8}. Thus, we have partially constructed this graph (Fig. 5), and it remains to determine the subgraph induced by the vertices $u_5, \dots, u_{12}, v_5, \dots, v_{12}$.

Let $U_1 = \{u_5, \dots, u_8\}$, $U_2 = \{u_9, \dots, u_{12}\}$, $V_1 = \{v_5, \dots, v_8\}$ and $V_2 = \{v_9, \dots, v_{12}\}$. Because of the absence of quadrilaterals, we have that each vertex of U_k $(k = 1, 2)$ is adjacent to exactly one vertex of V_l $(l = 1, 2)$, and that i^{th} $(i = 1, \dots, 4)$ vertex of U_k cannot be adjacent to i^{th} vertex of V_l. We denote by G_{kl} the subgraph induced by sets of vertices U_k and V_l. From previous facts, each G_{kl} has one of the types $a, b, c, \dots, i$, shown in Figure 6. We say that G_{kl} is of type x if i^{th} vertex of U_k identifies with i^{th} vertex of upper group of vertices of x, and i^{th} vertex of V_l identifies with i^{th} vertex of lower group of vertices of x.

Let G_{11} be of type a and G_{12} of type b. Then the vertices u_1, u_5, v_6 and v_{10} form the quadrilateral, which is contradiction. Then we say that types a and b are *incompatible*, otherwise they are *compatible*. Checking all pairs of types for compatibility, we find that the set of compatible pairs for G_{k1} and G_{k2} $(k = 1, 2)$ (as well as for G_{1l} and G_{2l} $(l = 1, 2)$) is

$$P = \{\ \{a,d\}, \{a,e\}, \{a,g\}, \{a,h\}, \{b,d\}, \{b,f\},\\ \{c,h\}, \{c,i\}, \{d,f\}, \{d,h\}, \{e,g\}, \{h,i\}\}.$$



D_{16}

Figure 7: Integral graph with spectrum $4, 2^{10}, 1^4, -1^4, -2^{10}, -4$

Now if $G_{11} = a$ and $G_{12} = d$, then $G_{21} \in \{d, e, g, h\}$, $G_{22} \in \{a, b, h\}$, and possible compatible pairs for G_{21} and G_{22} are (d, a), (d, b), (d, f), (e, a), (g, a), (h, a). Continuing in this fashion we get all the possible pairs for G_{21} and G_{22}. There is a total of 60 possibilities for the subgraph H induced by the vertices of $U_1 \cup U_2 \cup V_1 \cup V_2$. Eliminating those containing quadrilaterals, 24 possibilities remain, and only 6 of these give a new integral graph D_{16} shown in Figure 7.

Thus, we have proved

Theorem 6. *The only graph with spectrum* $4, 2^{10}, 1^4, -1^4, -2^{10}, -4$ *is* D_{16}.

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Department of Mathematics,
Faculty of Science and Mathematics,
Višegradska 33,
18000 Niš, Serbia and Montenegro
E–mail: dragance@pmf.ni.ac.yu

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